

Some Considerations About Podolsky-Axionic Electrodynamics

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Abstract

For a Podolsky-axionic electrodynamics, we compute the interaction potential within the structure of the gauge-invariant but path-dependent variables formalism. The result is equivalent to that of axionic electrodynamics from a new noncommutative approach, up to first order in θ .

1 Introduction

Higher order derivative theories have been discussed in the literature by a large number of authors [1,2,3,4,5,6,7,8,9,10,11], mainly due to the possibility of obtaining finite theories at short distances. An illustrative example of such a class of theories is the electrodynamics proposed by Podolsky [2], i.e. the $U(1)$ gauge theory where a quadratic term in the divergence of the field strength tensor is added to the free Lagrangian of the $U(1)$ sector. As a result, this new theory is endowed with interesting features such as a finite electron self-energy and a regular point charge electric field at the origin. In this context it may be recalled that field theories with higher derivatives have also attracted considerable attention in connection with supersymmetric [12] and string theories [13]. Another interesting theory is the electrodynamics proposed by Lee and Wick [14,15], i.e. the $U(1)$ gauge theory where a dimensional-6 operator containing higher derivatives is added to the free Lagrangian of the $U(1)$ sector. In this connection we call attention to the fact that, following this proposal, recently a great deal of attention has been devoted to the study of modifications to the Standard Model which stabilizes the Higgs mass against quadratically divergent corrections leading to the Lee-Wick Standard Model [16]. More recently, by using a novel way to formulate noncommutative quantum field theory (or

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quantum field theory in the presence of a minimal length) [17,18,19] we have obtained an ultraviolet finite electrodynamics [20], where the cutoff is provided by the noncommutative parameter θ .

On the other hand, we also point out that recently considerable attention has been paid to the formulation and experimental consequences of extensions of the Standard Model such as axion-like particles. Mention should be made, at this point, to axionic electrodynamics which experiences mass generation due to the breaking of rotational invariance induced by a classical background configuration of the gauge field strength [21]. Interestingly enough, we mention that axionic electrodynamics leads to a confining potential in the presence of a nontrivial constant expectation values for the gauge field strength $F_{\mu\nu}$ [22].

In this perspective, and given the recent interest in higher order derivative theories, it is instructive to further explore the physical consequences presented by this class of theories. Specifically, in this work we will focus attention on the impact of these higher-order terms on physical observables, in particular the static potential between two charges, using the gauge-invariant but path-dependent variables formalism, which provides a physically-based alternative to the Wilson loop approach [23,24,25,26]. In fact, our analysis leads to an ultraviolet finite static potential for axionic electrodynamics which is the sum of a Yukawa-type and a linear potential, leading to the confinement of static charges. Incidentally, the above static potential profile is equivalent to that of our noncommutative axionic electrodynamics up to first order in θ . In this way we establish a connection between noncommutative and Podolsky electrodynamics, Although a preliminary analysis about these issues has appeared before [20,27], we think is of value to clarify them because, in our view, have not been properly emphasized. In addition, the above connections are of interest from the point of view of providing unifications among diverse models. The paper is organized as follows. In the subsequent section, we present a brief review on Podolsky electrodynamics, for which we compute the static potential by two distinct methods. We show their equivalence and we set up the background for the Section III. In Section III, we consider the Podolsky-axionic electrodynamics and we show that a confining potential is achieved in the regime the axionic degree of freedom decouples. A summary of our work is the subject of our final section.

2 Brief review on Podolsky electrodynamics

We now reexamine the interaction energy between static pointlike sources for Podolsky electrodynamics, through two different methods. The first approach is based on the path-integral approach, whereas the second one makes use of the gauge-invariant but path-dependent variables formalism. The initial point

of our analysis is the Lagrangian density:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{l^2}{2}\left(\partial_\alpha F^{\beta\alpha}\right)^2 - A_\mu J^\mu, \quad (1)$$

which can also be written as

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}\left(1 + l^2\Delta\right)F^{\mu\nu} - A_\mu J^\mu, \quad (2)$$

where l is a constant with dimension (-1) in mass units, $\Delta \equiv \partial_\mu \partial^\mu$ and J_μ is an external current. One immediately sees that the Lagrangian Eq. (1) (or (2)) contains second order time derivatives of the potentials. This point motivates us to study the role of higher order derivatives on a physical observable.

Let us start off our considerations by computing the interaction energy between static pointlike sources for Podolsky electrodynamics via a path-integral approach. For this purpose, we begin by writing down the functional generator of the Green's functions, that is,

$$Z[J] = \exp\left(-\frac{i}{2}\int d^4x d^4y J^\mu(x) D_{\mu\nu}(x, y) J^\nu(y)\right), \quad (3)$$

where $D_{\mu\nu}(x, y) = \int \frac{d^4k}{(2\pi)^4} D_{\mu\nu}(k) e^{-ikx}$ is the propagator in the Feynman gauge. In this case, the corresponding propagator is given by

$$D_{\mu\nu}(k) = -\frac{1}{k^2(1 - l^2k^2)} \left\{ \eta_{\mu\nu} - \left[1 - (1 - l^2k^2)\right] \frac{k_\mu k_\nu}{k^2} \right\}. \quad (4)$$

By means of expression $Z = e^{iW[J]}$, and employing Eq. (3), $W[J]$ takes the form

$$W[J] = -\frac{1}{2}\int \frac{d^4k}{(2\pi)^4} J_\mu^*(k) \left[-\frac{1}{k^2(1 - l^2k^2)} \eta^{\mu\nu} + \frac{[1 - (1 - l^2k^2)]}{k^2(1 - l^2k^2)} \frac{k^\mu k^\nu}{k^2} \right] J_\nu(k). \quad (5)$$

Since the current $J^\mu(k)$ is conserved, expression (5) then becomes

$$W[J] = \frac{1}{2}\int \frac{d^4k}{(2\pi)^4} J_\mu^*(k) \frac{1}{k^2(1 - l^2k^2)} J^\mu(k). \quad (6)$$

Next, for $J_\mu(x) = [Q\delta^{(3)}(\mathbf{x} - \mathbf{x}^{(1)}) + Q'\delta^{(3)}(\mathbf{x} - \mathbf{x}^{(2)})]\delta_\mu^0$, and using standard functional techniques [28,29], we obtain that the interaction energy of the system is given by

$$U(r) = QQ' \int \frac{d^3k}{(2\pi)^3} \frac{m^2}{\mathbf{k}^2(\mathbf{k}^2 + m^2)} e^{i\mathbf{k}\cdot\mathbf{r}}, \quad (7)$$

where $\mathbf{r} \equiv \mathbf{x}^{(1)} - \mathbf{x}^{(2)}$ and $m^2 = 1/l^2$. This, together with $Q' = -Q$, yields finally

$$U(r) = -\frac{Q^2}{4\pi r} (1 - e^{-mr}), \quad (8)$$

with $r = |\mathbf{r}|$. From this expression it should be clear that the interaction energy is regular at the origin, in contrast to the usual Maxwell theory. In this respect the above result clearly shows the key role played by the "regularized propagator" in Eq. (6).

We shall now calculate the static potential using this time a gauge-invariant but path-dependent variables formalism along the lines of Refs. [23,24,25,26]. To this end, we will compute the expectation value of the energy operator H in the physical state $|\Phi\rangle$ describing the sources, which we will denote by $\langle H \rangle_\Phi$. In such a case, to obtain the corresponding Hamiltonian, we must carry out the quantization of the theory. Before going into details, we recall that the system described by (1) contains second order derivatives, hence to construct the Hamiltonian one must consider the velocities as independent canonical variables. Thus, the phase-space coordinate for the theory under consideration is given by $(A_\mu, \Pi^\nu) \oplus (\dot{A}_\mu, \Pi^{(1)\nu})$, where $\Pi^{(1)\nu}$ is the canonical momentum conjugate to \dot{A}_μ . This consideration implies that the canonical Hamiltonian H_C takes the form

$$H_C = \int d^3x \left(\Pi_\mu \dot{A}^\mu + \Pi_\mu^{(1)} \ddot{A}^\mu - \mathcal{L} \right). \quad (9)$$

In this case, the momenta are given by:

$$\Pi_\mu = F_{\mu 0} - l^2 \left[\partial^\nu \dot{F}_{\nu\mu} + \eta_{\mu 0} \partial^i \partial^\nu F_{i\nu} + \partial_\mu \partial^\nu F_{0\nu} \right], \quad (10)$$

and

$$\Pi_\mu^{(1)} = -l^2 \left[\partial^\nu F_{\mu\nu} + \eta_{\mu 0} \partial^\nu F_{\nu 0} \right]. \quad (11)$$

It is not hard to check that, in the electrostatic case ($\dot{\mathbf{E}} = 0$ and $\mathbf{B} = 0$), $\Pi_0^{(1)} = 0$, $\Pi_i^{(1)} = 0$ and $\Pi_0 = 0$. Therefore, in the electrostatic case under consideration, the canonical Hamiltonian is computed via the standard Legendre transformation. Accordingly, the canonical Hamiltonian reduces to

$$H_C = \int d^3x \left\{ \Pi^i \partial_i A_0 - \frac{1}{2} \Pi^i \left(1 - \frac{\Delta}{\Delta + m^2} \right) \Pi_i + \frac{1}{4} F_{ij} \frac{(\Delta + m^2)}{m^2} F^{ij} \right\}. \quad (12)$$

Notice that, for notational convenience, we have maintained Δ in (12). As already explained, in the electrostatic case under consideration Δ can be replaced by $-\nabla^2$, without any problem. Next, requiring the primary constraint Π_0 to be stationary, leads to the secondary constraint, $\Gamma_1(x) \equiv \partial_i \Pi^i = 0$. It is straightforward to check that there are no further constraints in the theory. Consequently, the extended Hamiltonian that generates translations in time then reads $H = H_C + \int d^3x (c_0(x) \Pi_0(x) + c_1(x) \Gamma_1(x))$. Here $c_0(x)$ and

$c_1(x)$ are arbitrary Lagrange multipliers. Moreover, it follows from this Hamiltonian that $\dot{A}_0(x) = [A_0(x), H] = c_0(x)$, which is an arbitrary function. Since $\Pi^0 = 0$ always, neither A^0 nor Π^0 are of interest in describing the system and may be discarded from the theory. The Hamiltonian is then

$$H = \int d^3x \left\{ c(x) \partial_i \Pi^i - \frac{1}{2} \Pi^i \left(1 - \frac{\Delta}{\Delta + m^2} \right) \Pi_i + \frac{1}{4} F_{ij} \frac{(\Delta + m^2)}{m^2} F^{ij} \right\}, \quad (13)$$

where $c(x) = c_1(x) - A_0(x)$.

We can at this stage impose a gauge condition, so that in conjunction with the constraint $\Pi^0 = 0$, it is rendered into a second class set. A particularly convenient choice is

$$\Gamma_2(x) \equiv \int_{C_{\xi x}} dz^\nu A_\nu(z) \equiv \int_0^1 d\lambda x^i A_i(\lambda x) = 0, \quad (14)$$

where λ ($0 \leq \lambda \leq 1$) is the parameter describing the spacelike straight path $x^i = \xi^i + \lambda(x - \xi)^i$, and ξ is a fixed point (reference point). There is no essential loss of generality if we restrict our considerations to $\xi^i = 0$. The choice (14) leads to the Poincaré gauge [30,31]. The Dirac brackets can now be determined and we simply note the only nontrivial Dirac bracket involving the field variables,

$$\{A_i(x), \Pi^j(y)\}^* = \delta_i^j \delta^{(3)}(x - y) - \partial_i^x \int_0^1 d\lambda x^j \delta^{(3)}(\lambda x - y). \quad (15)$$

Our next task is to compute the interaction energy. As mentioned before, to do that we need to calculate the expectation value of the energy operator H in the physical state $|\Phi\rangle$. Following Dirac [32], we write the physical state $|\Phi\rangle$ as

$$|\Phi\rangle \equiv |\bar{\Psi}(\mathbf{y}) \Psi(\mathbf{y}')\rangle = \bar{\psi}(\mathbf{y}) \exp \left(iq \int_{\mathbf{y}'}^{\mathbf{y}} dz^i A_i(z) \right) \psi(\mathbf{y}') |0\rangle, \quad (16)$$

where the line integral is along a spacelike path on a fixed time slice, q is the fermionic charge, and $|0\rangle$ is the physical vacuum state.

Taking the above Hamiltonian structure into account, we see that

$$\Pi_i(x) |\bar{\Psi}(\mathbf{y}) \Psi(\mathbf{y}')\rangle = \bar{\Psi}(\mathbf{y}) \Psi(\mathbf{y}') \Pi_i(x) |0\rangle + q \int_{\mathbf{y}}^{\mathbf{y}'} dz_i \delta^{(3)}(\mathbf{z} - \mathbf{x}) |\Phi\rangle.$$

Therefore, $\langle H \rangle_\Phi$ can be written as

$$\langle H \rangle_\Phi = \langle H \rangle_0 + \langle H \rangle_\Phi^{(1)} + \langle H \rangle_\Phi^{(2)}, \quad (17)$$

where $\langle H \rangle_0 = \langle 0 | H | 0 \rangle$. The $\langle H \rangle_\Phi^{(1)}$ and $\langle H \rangle_\Phi^{(2)}$ terms are given by

$$\langle H \rangle_\Phi^{(1)} = -\frac{1}{2} \langle \Phi | \int d^3x \Pi_i \Pi^i | \Phi \rangle, \quad (18)$$

and

$$\langle H \rangle_\Phi^{(2)} = \frac{1}{2} \langle \Phi | \int d^3x \Pi_i \frac{\nabla^2}{(\nabla^2 - m^2)} \Pi^i | \Phi \rangle. \quad (19)$$

Using Eq. (17), and following our earlier procedure [23,24,25,26], we see that the potential for two opposite charges, located at \mathbf{y} and \mathbf{y}' , takes the form

$$U(r) = -\frac{q^2}{4\pi r} (1 - e^{-mr}), \quad (20)$$

where $|\mathbf{y} - \mathbf{y}'| = r$. It is worth noting that these approaches, despite being completely different, lead to the same result which seems to indicate that they are equivalent term by term. It should, however, be noted here that the central difference between the above analysis and that leading to Eq. (8) rests in the fact that the potential (20) is directly recovered from the constraints structure of the theory we have discussed.

One can now further observe that there is an alternative but equivalent way of obtaining the result [23,30,31], which highlights certain distinctive features of our methodology. In order to illustrate the discussion, we start by observing that:

$$U \equiv q (\mathcal{A}_0(\mathbf{0}) - \mathcal{A}_0(\mathbf{y})), \quad (21)$$

where the physical scalar potential is given by

$$\mathcal{A}_0(x^0, \mathbf{x}) = \int_0^1 d\lambda x^i E_i(\lambda \mathbf{x}), \quad (22)$$

with $i = 1, 2, 3$. This follows from the vector gauge-invariant field expression:

$$\mathcal{A}_\mu(x) \equiv A_\mu(x) + \partial_\mu \left(-\int_\xi^x dz^\mu A_\mu(z) \right), \quad (23)$$

where, as in Eq. (16), the line integral is along a spacelike path from the point ξ to x , on a fixed slice time. The gauge-invariant variables (23) commute with the sole first constraint (Gauss' law), confirming in this way that these fields are physical variables. Note that Gauss' law for the present theory reads $\partial_i \Pi^i = J^0$, where we have included the external current J^0 to represent the presence of two opposite charges. For $J^0(t, \mathbf{x}) = q\delta^{(3)}(\mathbf{x})$ the electric field is given by

$$E^i = q\partial^i (G(\mathbf{x}) - G'(\mathbf{x})), \quad (24)$$

where $G(\mathbf{x}) = \frac{1}{4\pi|\mathbf{x}|}$ and $G'(\mathbf{x}) = \frac{e^{-m|\mathbf{x}|}}{4\pi|\mathbf{x}|}$ are the Green functions in three space dimensions. Finally, replacing this result in (22) and using (21), we reobtain

Eqs. (8) and (20), i.e.,

$$U(r) = -\frac{q^2}{4\pi r} (1 - e^{-mr}). \quad (25)$$

Notice that the procedure leading to Eq. (8), compared with the above one, involves the gauge field propagator, which leads us to conclude that the contributions of the propagator are properly captured in the gauge-invariant variables formalism. This concludes our considerations about Podolsky electrodynamics.

3 Podolsky-Axionic electrodynamics

As already stated, our next undertaking is to use the ideas of the previous section in order to consider Podolsky-axionic electrodynamics. In such a case the Lagrangian density reads

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu} (1 + l^2\Delta) F^{\mu\nu} + \frac{1}{2}(\partial_\mu\varphi)^2 - \frac{1}{2}\mu^2\varphi^2 + \frac{\lambda}{4}\varphi\tilde{F}^{\mu\nu}F_{\mu\nu}. \quad (26)$$

Before we proceed to work out explicitly the energy, let us commence our considerations with a short presentation of previous results stemming from the gauge-invariant formalism [23,24,25,26]. For this purpose, we carry out the integration over the φ -field. Furthermore, as was explained in [22], by considering static scalar fields we may replace $\Delta\varphi = -\nabla^2\varphi$. In this case, the effective theory takes the form

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu} (1 + l^2\Delta) F^{\mu\nu} - \frac{\lambda^2}{32}(\tilde{F}_{\mu\nu}F^{\mu\nu}) \frac{1}{\nabla^2 - \mu^2}(\tilde{F}_{\alpha\beta}F^{\alpha\beta}). \quad (27)$$

Following our earlier discussion, after splitting $F_{\mu\nu}$ in the sum of a classical background, $\langle F_{\mu\nu} \rangle$, and a small fluctuation, $f_{\mu\nu}$, the Lagrangian (27) up to quadratic terms in the fluctuations then becomes

$$\mathcal{L} = -\frac{1}{4}f_{\mu\nu} (1 + l^2\Delta) f^{\mu\nu} - \frac{\lambda^2}{32}v^{\mu\nu}f_{\mu\nu} \frac{1}{\nabla^2 - \mu^2}v^{\lambda\rho}f_{\lambda\rho}, \quad (28)$$

where $f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$, a_μ stands for the fluctuation, and $\varepsilon^{\mu\nu\alpha\beta} \langle F_{\alpha\beta} \rangle \equiv v^{\mu\nu}$ and $\varepsilon^{\rho\lambda\gamma\delta} \langle F_{\gamma\delta} \rangle \equiv v^{\rho\lambda}$.

After having obtained the general effective theory, we now turn our attention to the calculation of the interaction energy in the $v^{0i} \neq 0$ and $v^{ij} = 0$ case (referred to as the electric one in what follows). In such a case, the Lagrangian

(28) reduces to

$$\mathcal{L} = -\frac{1}{4}f_{\mu\nu} \left(1 + l^2\Delta\right) f^{\mu\nu} - \frac{\lambda^2}{32}v^{0i}f_{0i}\frac{1}{\nabla^2 - \mu^2}v^{0k}f_{0k}. \quad (29)$$

It is now once again straightforward to apply the formalism discussed in the preceding section. Therefore the canonical Hamiltonian is

$$H_C = \int d^3x \left\{ \Pi_i \partial^i a^0 + \frac{1}{2} \Pi^i \left[\frac{\nabla^2 - \mu^2}{(1 + l^2\Delta)(\nabla^2 - \mu^2) - w^2} \right] \Pi^i + \frac{1}{2} \mathbf{B}^2 \right\}, \quad (30)$$

where $w^2 = \frac{\lambda^2}{4}\mathcal{B}^2$. Here \mathbf{B} and \mathcal{B} represent the magnetic field fluctuation and external (background) magnetic field.

By means of expression (17) we write the expectation value as

$$\langle H \rangle_\Phi = \langle H \rangle_0 + \langle H \rangle_\Phi^{(1)} + \langle H \rangle_\Phi^{(2)}, \quad (31)$$

with $\langle H \rangle_0 = \langle 0 | H | 0 \rangle$, while the terms $\langle H \rangle_\Phi^{(1)}$ and $\langle H \rangle_\Phi^{(2)}$ are given by

$$\begin{aligned} \langle H \rangle_\Phi^{(1)} &= -\frac{1}{2} \frac{1}{\sqrt{1 - 2l^2(\mu^2 + 2w^2) + \mu^4 l^4}} \\ &\times \langle \Phi | \int d^3x \Pi_i \left\{ \frac{\nabla^2}{(\nabla^2 - M_2^2)} - \frac{\nabla^2}{(\nabla^2 - M_1^2)} \right\} \Pi^i | \Phi \rangle, \end{aligned} \quad (32)$$

and

$$\begin{aligned} \langle H \rangle_\Phi^{(2)} &= \frac{1}{2} \frac{1}{\sqrt{1 - 2l^2(\mu^2 + 2w^2) + \mu^4 l^4}} \\ &\times \langle \Phi | \int d^3x \Pi_i \left\{ \frac{1}{(\nabla^2 - M_1^2)} - \frac{1}{(\nabla^2 - M_2^2)} \right\} \Pi^i | \Phi \rangle. \end{aligned} \quad (33)$$

Here $M_1^2 = \frac{1}{2l^2} \left[(1 + \mu^2 l^2) + \sqrt{1 - 2l^2(\mu^2 + 2w^2) + \mu^4 l^4} \right]$ and $M_2^2 = \frac{1}{2l^2} \left[(1 + \mu^2 l^2) - \sqrt{1 - 2l^2(\mu^2 + 2w^2) + \mu^4 l^4} \right]$, where $|1 - \mu^2 l^2| > 2\lambda\omega$, which ensures $M_1^2 > 0$ and $M_2^2 > 0$.

Accordingly, the potential for a pair of point-like opposite charges q located at $\mathbf{0}$ and \mathbf{L} takes the form

$$\begin{aligned}
U = & -\frac{q^2}{4\pi} \frac{1}{\sqrt{1 - 2l^2(\mu^2 + 2w^2) + \mu^4 l^4}} \left[\frac{e^{-M_2 L}}{L} - \frac{e^{-M_1 L}}{L} \right] \\
& + \frac{q^2}{8\pi} \frac{\mu^2}{\sqrt{1 - 2l^2(\mu^2 + 2w^2) + \mu^4 l^4}} \ln \left(\frac{M_2^2}{M_1^2} \right) L.
\end{aligned} \tag{34}$$

Expression (34) immediately shows that the effect of including higher order derivative terms is an ultraviolet finite static potential, which is the sum of a Yukawa and a linear potential, leading to the confinement of static charges. Another interesting finding is the presence of a finite string tension in Eq. (34). Evidently, this improves the analysis as compared to our previous studies [22,33], where an ultraviolet cutoff has been introduced by hand.

4 Final Remarks

Let us summarize our work. Using the gauge-invariant but path-dependent formalism, we have computed the static potential for Podolsky-axionic electrodynamics. Interestingly, we have obtained an ultraviolet finite static potential, which is the sum of a Yukawa-type and a linear potential, leading to the confinement of static charges. As already expressed, the above static potential profile is equivalent to that of our noncommutative axionic electrodynamics up to first order in θ . In this way we have provided a new connection between effective models. Accordingly, the benefit of considering the present framework is to provide unifications among different models, as well as exploiting the equivalence in explicit calculations, as we have seen in the course of the present discussion. Finally, an explicit expression of the effective potential between two static charges could be of interest for searching for bounds of the constant l .

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